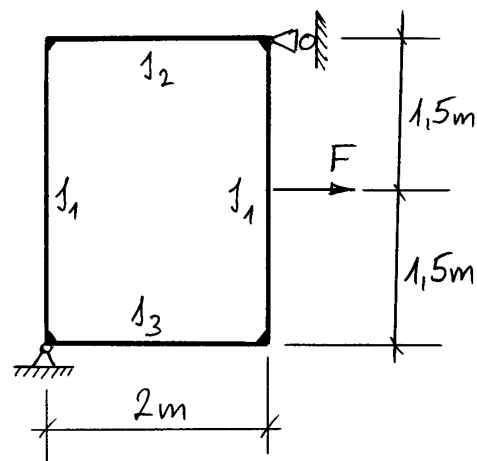


**1. Feladat (25 pont):**

- a) Határozza meg a vázolt zárt keret hajlító igénybevételi ábráját  $\sigma$ -ponti módszerrel!

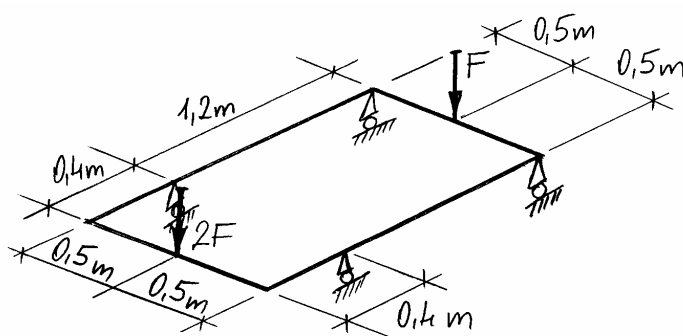
Adatok:  $F = 500 \text{ N}$ ;  $I_2 = 6 \cdot I_1$ ;  $I_3 = \frac{2}{3} \cdot I_1$



**2. Feladat (25 pont):** A vázolt alvázkeretet két helyen terheli erő. Az alvázat alkotó minden gerenda hajlításra vett másodrendű nyomatéka  $I$ , és a torziós másodrendű nyomatéka  $I_t$ .

- a) Határozza meg a keret hajlító és csavaró igénybevételi ábráit!

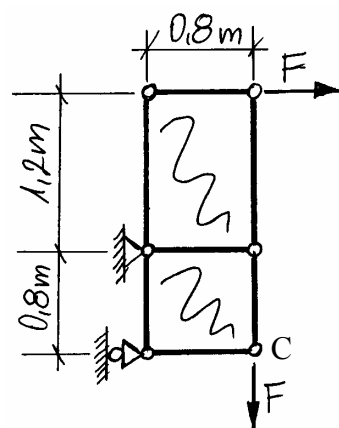
Adatok:  $F = 4 \text{ kN}$ ;  $I_t = 1,3 \cdot I$ ;  $\nu = 0,3$



**3. Feladat (25 pont):** A vázolt lemezzel merevített szerkezetet két helyen terheli  $F$  nagyságú erő. A rudak keresztmetszeti területe  $A_k$ , a lemez vastagsága  $v$ .

- a) Határozza meg a szerkezet nyírófolyam és normálerő ábráját!  
 b) Határozza meg a C pont függőleges irányú elmozdulásának ( $y_C$ ) nagyságát és irányát!

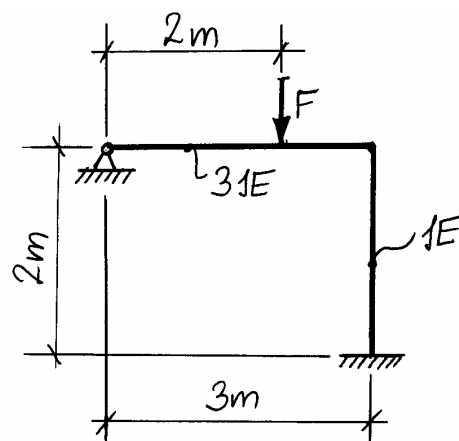
Adatok:  $F = 600 \text{ N}$ ;  $E = 200 \text{ GPa}$ ;  $A_k = 200 \text{ mm}^2$ ;  $G = 80 \text{ GPa}$ ;  
 $\nu = 0,8 \text{ mm}$

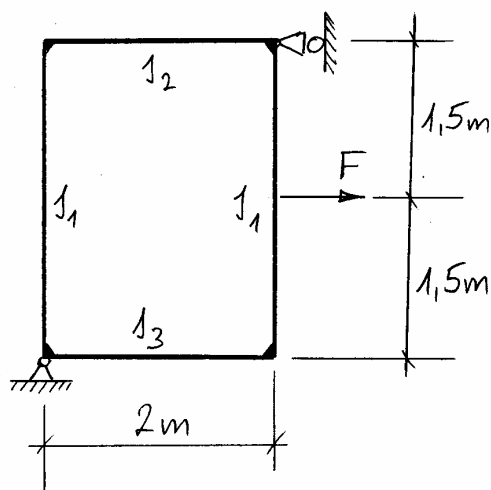


**4. Feladat (25 pont):** A vázolt tartó gerendáinak húzó-nyomó merevsége végtelen.

- a) Határozza meg a tartó hajlító igénybevételi ábráját!

Adatok:  $F = 900 \text{ N}$ ;  $AE = \infty$



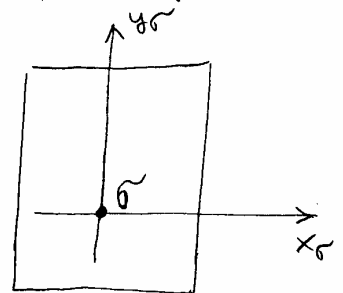
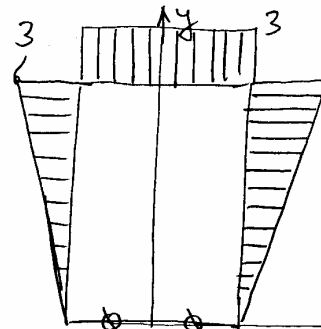


$$F = 500 \text{ N}$$

$$I_2 = 6 \cdot I_1$$

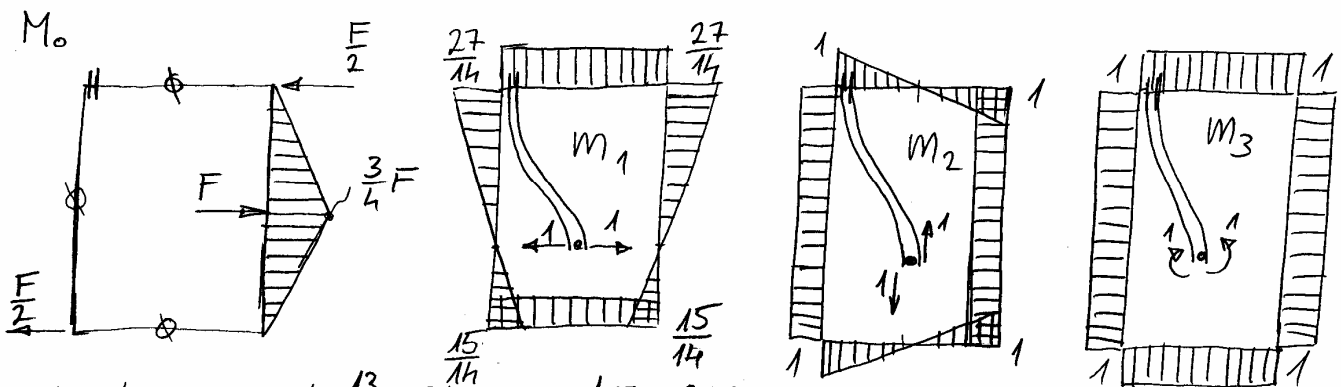
$$I_3 = \frac{2}{3} \cdot I_1$$

Szimmetria  
matt ezre a  
bírálások!



$$\bar{y}_G = \frac{\int \frac{y}{I_E} ds}{\int \frac{ds}{I_E}} = \frac{\frac{1}{I_1 E} \left[ \frac{3 \cdot 3}{2} \right] + \frac{1}{I_2 E} [3 \cdot 2] + \frac{1}{I_1 E} \left[ \frac{3 \cdot 3}{2} \right] + \frac{1}{I_3 E} [0]}{\frac{1}{I_1 E} \cdot 3 + \frac{1}{I_2 E} \cdot 2 + \frac{1}{I_1 E} \cdot 3 + \frac{1}{I_3 E} \cdot 2} =$$

$$= \frac{\frac{1}{I_1 E} (4.5 + 1 + 4.5 + 0)}{\frac{1}{I_1 E} (3 + \frac{1}{3} + 3 + 3)} = \frac{30}{28} = 1.0714 \text{ m}$$



$$\sigma_{10} = \frac{1}{I_1 E} \left[ \frac{3}{4} F \cdot 1.5 \cdot \frac{1}{2} \cdot \frac{13}{14} + \frac{3}{4} F \cdot 1.5 \cdot \frac{1}{2} \cdot -\frac{1}{14} \right] = \frac{0.4821 F}{I_1 E}$$

$$\sigma_{20} = \frac{1}{I_1 E} \left[ \frac{3}{4} F \cdot 1.5 \cdot \frac{1}{2} \cdot -1 \right] \cdot 2 = -\frac{1.125 F}{I_1 E}$$

$$\sigma_{30} = \frac{1}{I_1 E} \left[ \frac{3}{4} F \cdot 1.5 \cdot \frac{1}{2} \cdot 1 \right] \cdot 2 = \frac{1.125 F}{I_1 E}$$

$$\sigma_{11} = \frac{1}{I_1 E} \left[ \frac{1.929 \cdot 1.929}{2} \cdot \frac{2}{3} \cdot 1.929 + \frac{1.071 \cdot 1.071}{2} \cdot \frac{2}{3} \cdot 1.071 \right] \cdot 2 + \frac{1}{I_2 E} [1.929 \cdot 2 \cdot 1.929] + \frac{1}{I_3 E} [1.071 \cdot 2 \cdot 1.071] = \frac{9.428}{I_1 E}$$

$$\sigma_{22} = \frac{1}{I_1 E} [1 \cdot 3 \cdot 1] \cdot 2 + \frac{1}{I_2 E} \left[ \frac{1 \cdot 1}{2} \cdot \frac{2}{3} \cdot 1 \right] \cdot 2 + \frac{1}{I_3 E} \left[ \frac{1 \cdot 1}{2} \cdot \frac{2}{3} \cdot 1 \right] \cdot 2 = \frac{7.33}{I_1 E}$$

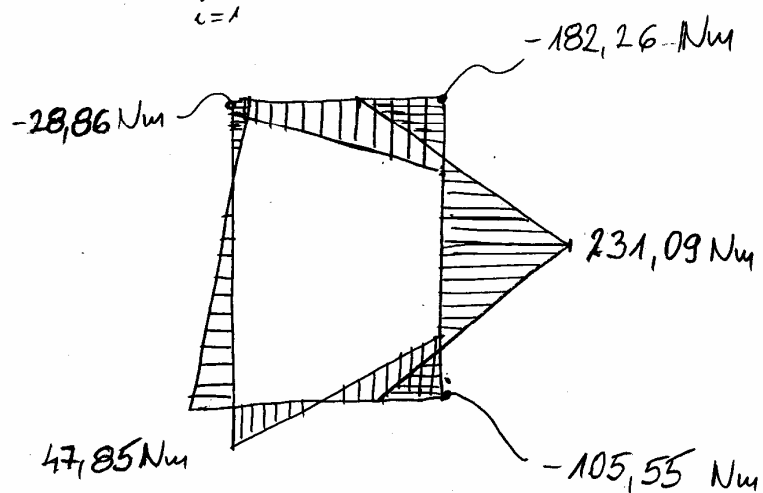
$$\sigma_{33} = \frac{1}{I_1 E} [1 \cdot 3 \cdot 1] \cdot 2 + \frac{1}{I_2 E} [1 \cdot 2 \cdot 1] + \frac{1}{I_3 E} [1 \cdot 2 \cdot 1] = \frac{10}{I_1 E}$$

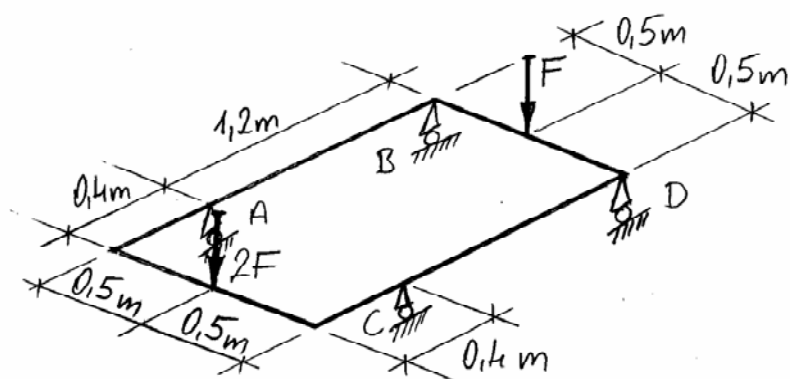
$$x_1 = -\frac{\sigma_{10}}{\sigma_{11}} = -0.0511 \cdot F = -25.57 \text{ N}$$

$$x_3 = -\frac{\sigma_{30}}{\sigma_{33}} = -0.1125 F = -56.25 \text{ N}$$

$$x_2 = -\frac{\sigma_{20}}{\sigma_{22}} = 0.1534 \cdot F = 76.7 \text{ N}$$

$$M = M_0 + \sum_{i=1}^3 x_i \cdot m_i$$





$$F = 400 \text{ N}$$

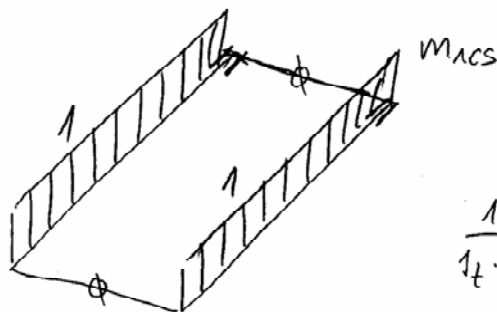
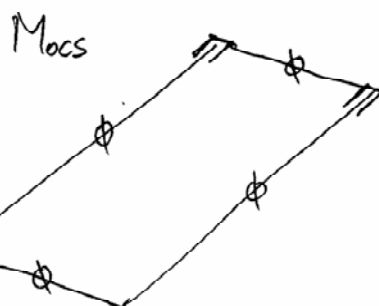
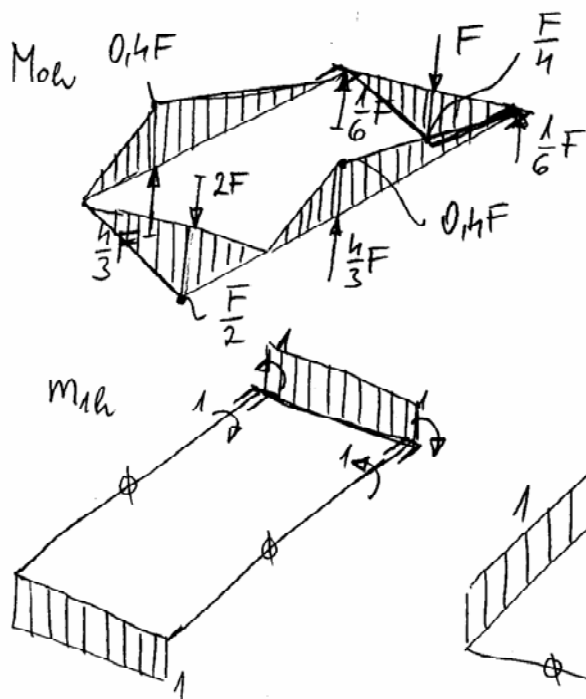
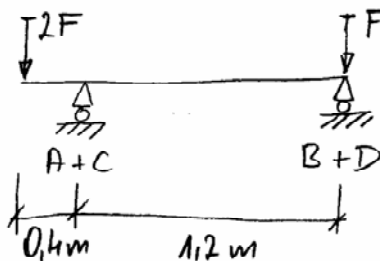
$$I_t = 1,3 \cdot I$$

$$\nu = 0,3$$

$$G = \frac{E}{2(1+\nu)} = \frac{E}{2,6}$$

$$A = \frac{4}{3}F \quad B = \frac{1}{6}F$$

$$C = \frac{4}{3}F \quad D = \frac{1}{6}F$$

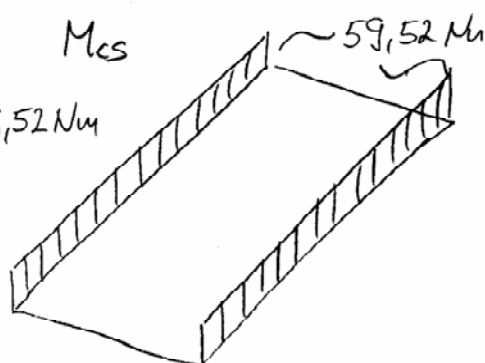
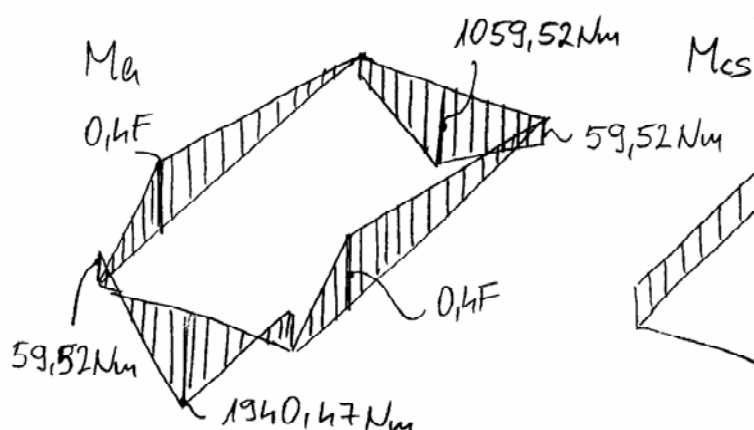


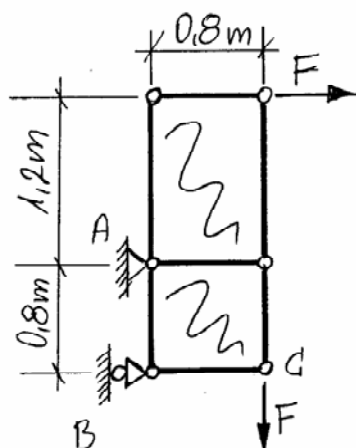
$$\frac{1}{I_t G} = \frac{1}{1,3 \cdot I \cdot \frac{E}{2,6}} = \frac{2}{1E}$$

$$\tilde{\sigma}_{10} = \frac{1}{1E} \left[ \frac{F}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{F}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot -1 \right] \cdot 2 + \frac{2}{1E} [0] = \frac{1}{1E} \cdot \frac{F}{8} = \frac{500}{1E}$$

$$\tilde{\sigma}_{11} = \frac{1}{1E} [1 \cdot 1 \cdot 1] \cdot 2 + \frac{2}{1E} [1 \cdot 1 \cdot 6 \cdot 1] \cdot 2 = \frac{8,4}{1E}$$

$$\chi_1 = -\frac{\tilde{\sigma}_{10}}{\tilde{\sigma}_{11}} = -0,01488 F = -59,52 \text{ Nm}$$





$$F = 600 \text{ N}$$

$$E = 200 \text{ GPa}$$

$$G = 80 \text{ GPa}$$

$$A_k = 200 \text{ mm}^2$$

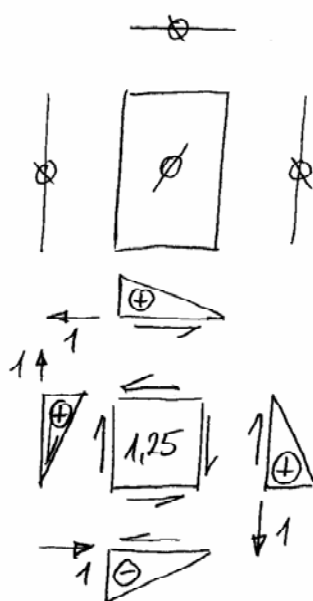
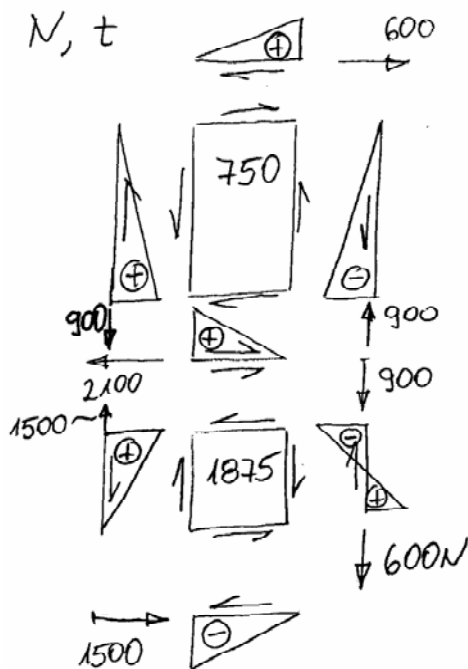
$$b = 0.8 \text{ mm}$$

$$B_x = 2.5 \cdot F = 1500 \text{ N} \rightarrow$$

$$A_x = 2100 \text{ N} \leftarrow = 3.5F$$

$$A_y = F = 600 \text{ N} \uparrow$$

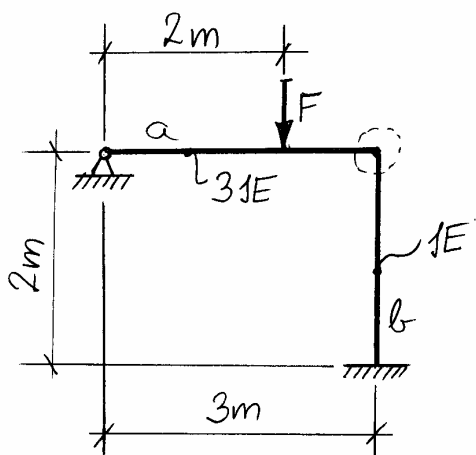
$N, t$



$$A_k \cdot E = 200 \cdot 10^{-6} \cdot 200 \cdot 10^9 = 4 \cdot 10^7 \text{ N}$$

$$G \cdot b = 80 \cdot 10^9 \cdot 0.8 \cdot 10^{-3} = 6.4 \cdot 10^7 \frac{\text{N}}{\text{m}}$$

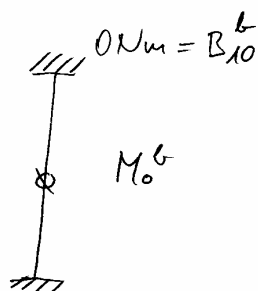
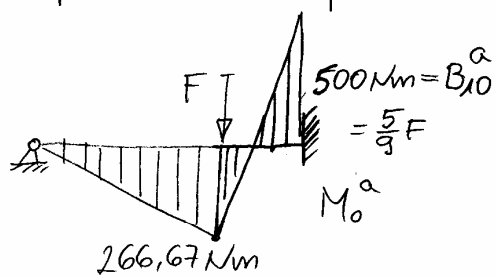
$$y_G = \frac{1}{A_k E} \left[ \frac{1500 \cdot 0.8}{2} \cdot \frac{2}{3} \cdot 1 + \frac{1 \cdot 0.8}{2} \cdot 100 + \frac{2100 \cdot 0.8}{2} \cdot \frac{2}{3} \cdot 1 + \frac{1500 \cdot 0.8}{2} \cdot \frac{2}{3} \cdot 1 \right] + \frac{1}{G \cdot b} [1875 \cdot 1.25 \cdot 0.8^2] = \frac{1400}{4 \cdot 10^7} + \frac{1500}{6.4 \cdot 10^7} = 1.156 \cdot 10^{-5} \text{ m} = 0.058 \text{ mm} \downarrow$$



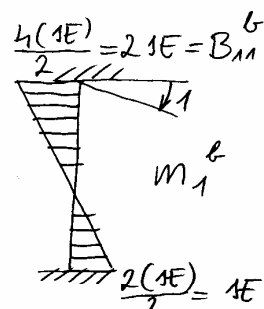
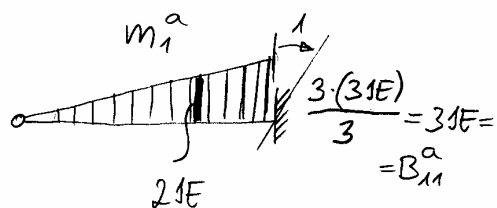
$$F = 900 \text{ N}$$

$$AE = \infty$$

1 szabadságfokú!



$$B_{10} = B_{10}^a + B_{10}^b = 500$$



$$B_{11} = B_{11}^a + B_{11}^b = 5EI$$

$$B_{10} + B_{11} \cdot \delta_1 = 0 \Rightarrow \delta_1 = -\frac{B_{10}}{B_{11}} = \frac{-500}{5EI} = \frac{-100}{EI} = \frac{-F}{3EI} \text{ [rad]}$$

$$M = M_0 + m_1 \cdot \delta_1$$

